全國公私立高級中學 107 學年度指定科目第七次聯合模擬考試

數學甲考科解析

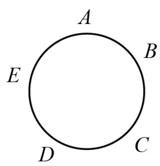
考試日期：108 年 5 月 7~8 日

1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	4	2	1	45	23	124	5	3	−	5	1	0	5
15	16												
8	3												

第壹部分：選擇題

一、單選題

1. ① 若有 0 人站著有 1 種情形
② 若有 1 人站著，有 5 種情形
③ 若有 2 人站著，有 (A,C)、(A,D)、(B,D)、(B,E)、(C,E)，5 種情形



$\therefore P=\dfrac{1+5+5}{2^5}=\dfrac{11}{32}$

故選(2)

2. $z_1-z_2=\dfrac{(2-4i)(2-i)}{(2+i)(2-i)}=\dfrac{4-10i-4}{4+1}=-2i$，$\therefore z_1,z_2$ 的實部相等

令 $z_1=a+bi,z_2=a+ci,b-c=-2$，

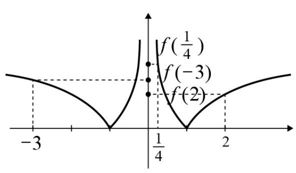
又 $|z_1|=|z_2|=1$，$a^2+b^2=1,a^2+c^2=1$

$(b+c)(b-c)=0,b+c=0,\therefore b=-1,c=1,a=0$

可知 $z_1=-i,z_2=i\Rightarrow z_1\cdot z_2=-i^2=1$

故選(4)

3. 由 $y=f(x)$ 對稱 y 軸，



$\therefore\ f(\dfrac{1}{4})>f(-3)>f(2)\Rightarrow b>a>c$

故選(2)

4. $x^2+y^2-2y=0\Rightarrow x^2+(y-1)^2=1$

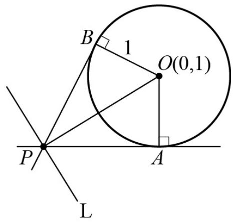
$\Rightarrow$ 圓心 $O(0,1)$，半徑 $r=1$

四邊形 $OAPB$ 最小面積$=2\Delta OAP$

$2=\overline{PA}\times1\Rightarrow\overline{PA}=2$，$\overline{PO}=\sqrt{5}$

最小值成立時發生在 $\overline{PO}$ 最小時

即 $\overline{OP}\perp L\Rightarrow d(O,L)=\dfrac{5}{\sqrt{k^2+1}}=\sqrt{5}$



$\therefore k^2+1=5,k^2=4,k=\pm2(\because \mathrm{k}>0,$負不合$)\therefore k=2$

故選(1)

二、多選題

5. (1) ×，令$L_1=\begin{cases}x=2t+1\\y=3t-1\\z=-t+2\end{cases}$，$t\in R$，$L_2=\begin{cases}x=-s+8\\y=2s+1\\z=s+8\end{cases}$，$s\in R$

$\Rightarrow\begin{cases}2t+1=-s+8-①\\3t-1=2s+1-②\\-t+2=s+8-③\end{cases}$

①＋③　$t=13$，$s=-19$，代入② $38\neq-37$

$\therefore$ L_1，L_2 歪斜

(2) ×，$\overrightarrow{n_E}=(1,1,-1)$，$\overrightarrow{d_2}=(-1,2,1)$

$\overrightarrow{n_E}\cdot\overrightarrow{d_2}=-1+2-1=0$

又將點$(8,1,8)$代入平面 E　　$8+1-8=1$

$\Rightarrow$ L_2 在平面上

(3) ×，$\overrightarrow{n_1}=\overrightarrow{d_1}\times\overrightarrow{d_3}=(5,-1,7)$

可得 E_1：$5x-y+7z=20$

(4) ○，$\overrightarrow{n_2}=\overrightarrow{d_1}\times\overrightarrow{d_2}=(5,-1,7)$

可得 E_2：$5x-y+7z=95$，$\therefore$ $E_2//E_1$

(5) ○，$\overline{PQ}=d\left((1,-1,2),E_2\right)=\dfrac{|5+1+14-95|}{\sqrt{25+1+49}}=\dfrac{75}{\sqrt{75}}=5\sqrt{3}$

故選(4)(5)

6. (1) ×，當 $f(x)$ 爲整係數方程式，且 $h,m\in Z,m\neq0$

(2) ○

(3) ○，$\deg f''(x)=3$，$f''(x)=0$ 最多有 3 個實根

$\Rightarrow$ 最多有 3 個反曲點

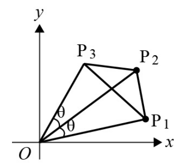
(4) ×，$\deg f'(x)=6,f'(x)=0$ 可能沒有實根

$\Rightarrow$ 可能沒有極值，反例：$f(x)=x^7$

(5) ×，反例：$f(x)=x^3$，$f'(x)=2x^2=0$，$x=0$，有實根

故選(2)(3)

7. $A=\begin{bmatrix}\frac{3}{5}&-\frac{4}{5}\\\frac{4}{5}&\frac{3}{5}\end{bmatrix}\Rightarrow\cos\theta=\dfrac{3}{5}$，$\sin\theta=\dfrac{4}{5}$



(1) ○，$\sin\angle P_1OP_3=\sin2\theta=2\left(\dfrac{4}{5}\right)\left(\dfrac{3}{5}\right)=\dfrac{24}{25}$

(2) ○，$\Delta P_1P_2P_3=\Delta OP_1P_2+\Delta OP_2P_3-\Delta OP_1P_3$

$=\dfrac{1}{2}k\times k\times\left(\dfrac{4}{5}\right)+\dfrac{1}{2}k\times k\times\left(\dfrac{4}{5}\right)-\dfrac{1}{2}k\times k\times\left(\dfrac{24}{25}\right)=\dfrac{8}{25}k^2$

(3) ×，$\overline{P_1P_3}^2=k^2+k^2-2\times k\times k\times\cos2\theta$

$\overline{P_1P_3}^2=\dfrac{64}{25}k^2\Rightarrow\overline{P_1P_3}=\dfrac{8}{5}k$

(4) ○，令$P_1(5t,\ 5t^2-5)$

$k^2=\overline{OP_1}^2=(5t)^2+(5t^2-5)^2$

$=25t^4-25t^2+25=25\left(t^2-\dfrac{1}{2}\right)^2+\dfrac{75}{4}$

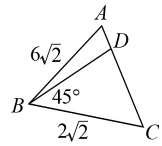
當$t^2=\dfrac{1}{2}$時，$\Delta P_1P_2P_3$面積最小值為$\dfrac{8}{25}\times\dfrac{75}{4}=6$

(5) ×，承(4)，此時$P_1\left(\pm\dfrac{5}{\sqrt{2}},-\dfrac{5}{2}\right)$

故選(1)(2)(4)

三、選填題

A. 由$\Delta ABC,\dfrac{2\sqrt{2}}{\sin A}=\dfrac{6\sqrt{2}}{\sin C}\Rightarrow\sin C=\dfrac{3}{\sqrt{10}}$



$\therefore\angle C$為銳角，$\therefore\cos C=\dfrac{1}{\sqrt{10}}$

又$\sin\angle BDC=\sin\left[180^\circ-(45^\circ+\angle C)\right]$

$=\sin(45^\circ+\angle C)$

$=\dfrac{1}{\sqrt{2}}\times\dfrac{1}{\sqrt{10}}+\dfrac{1}{\sqrt{2}}\times\dfrac{3}{\sqrt{10}}=\dfrac{4}{\sqrt{20}}=\dfrac{2}{\sqrt{5}}$

由$\Delta DBC,\dfrac{\overline{CD}}{\sin45^\circ}=\dfrac{2\sqrt{2}}{\sin\angle BDC}\Rightarrow\overline{CD}=\dfrac{2\sqrt{2}}{\frac{2}{\sqrt{5}}}\times\dfrac{1}{\sqrt{2}}=\sqrt{5}$

B. 當$|x|<1f(x)=\dfrac{ax^2+bx-3}{-5}$，當$|x|<1f(x)=\dfrac{ax^2+bx-3}{-5}$

$x=1$，$f(1)=\dfrac{a+b-2}{-4}$

$x=-1$，$f(-1)=\dfrac{a-b-4}{-4}$

$\because f(x)$ 爲連續函數

$\Rightarrow \begin{matrix}\lim\limits_{x\to 1^-} f(x)=f(1) \\ \lim\limits_{x\to -1^+} f(x)=f(-1)\end{matrix} \Rightarrow \begin{matrix}\dfrac{a+b-3}{-5}=\dfrac{a+b-2}{-4} \\ \dfrac{a-b-3}{-5}=\dfrac{a-b-4}{-4}\end{matrix}$

$\Rightarrow \begin{cases}a+b=-2 \\ a-b=8\end{cases}$，$\therefore a=3$，$b=-5 \ \Rightarrow (a,b)=(3,-5)$

C. 抽獎方式共有下列情形

①甲抽 3 次，$E_1=3\times 300\times \dfrac{C_2^2}{C_2^5}=90$

②甲抽 2 次，乙抽 1 次，$E_2=2\times 300\times \dfrac{C_2^2}{C_2^5}+150\times \dfrac{C_2^3}{C_2^5}=105$

③甲抽 1 次，乙抽 1 次，$E_3=1\times 300\times \dfrac{C_2^2}{C_2^5}+150\times \dfrac{C_2^3}{C_2^5}=75$

④乙抽 2 次，$E_4=2\times 150\times \dfrac{C_2^3}{C_2^5}=90$

∴獎金期望值最多爲 105 元

D. $\left|\overrightarrow{a}+2\overrightarrow{b}\right|=\left|\overrightarrow{a}\right|=2$

$\Rightarrow \left|\overrightarrow{a}\right|^2+4\overrightarrow{a}\cdot\overrightarrow{b}+4\left|\overrightarrow{b}\right|^2=\left|\overrightarrow{a}\right|^2$，$\therefore \overrightarrow{a}\cdot\overrightarrow{b}=-\left|\overrightarrow{b}\right|^2$

又$\left|2\overrightarrow{a}+\overrightarrow{b}\right|+\left|\overrightarrow{b}\right|$

$=\sqrt{\left|2\overrightarrow{a}+\overrightarrow{b}\right|^2}+\left|\overrightarrow{b}\right|$

$=\sqrt{4\left|\overrightarrow{a}\right|^2+4\overrightarrow{a}\cdot\overrightarrow{b}+\left|\overrightarrow{b}\right|^2}+\left|\overrightarrow{b}\right|=\sqrt{16-3\left|\overrightarrow{b}\right|^2}+\left|\overrightarrow{b}\right|$

又$16-3\left|\overrightarrow{b}\right|^2\ge 0, \therefore \left|\overrightarrow{b}\right|^2\le \dfrac{16}{3}, \therefore \left|\overrightarrow{b}\right|\le \dfrac{4}{\sqrt{3}}$

設$\left|\overrightarrow{b}\right|=\dfrac{4}{\sqrt{3}}\cos\theta, 0^\circ\le\theta\le 90^\circ$

則$\left|2\overrightarrow{a}+\overrightarrow{b}\right|+\left|\overrightarrow{b}\right|=\sqrt{16-3\times\dfrac{16}{3}\cos^2\theta}+\dfrac{4}{\sqrt{3}}\cos\theta$

$=4\sin\theta+\dfrac{4}{\sqrt{3}}\cos\theta$

$=\dfrac{8}{\sqrt{3}}\left(\dfrac{\sqrt{3}}{2}\sin\theta+\dfrac{1}{2}\cos\theta\right)$

$=\dfrac{8}{\sqrt{3}}\sin(\theta+30^\circ),$

當 $\sin(\theta+30^\circ)=1$ 時，$\theta+30^\circ=90^\circ$，$\theta=60^\circ$

$\left|2\overrightarrow{a}+\overrightarrow{b}\right|+\left|\overrightarrow{b}\right|$ 有最大值 $\dfrac{8}{\sqrt{3}}$

另解：

利用柯西不等式

$\left[\left(\sqrt{16-3\left|\overrightarrow{b}\right|^2}\right)^2+\left(\sqrt{3}\left|\overrightarrow{b}\right|\right)^2\right]\cdot\left[(1)^2+\left(\dfrac{1}{\sqrt{3}}\right)^2\right]$

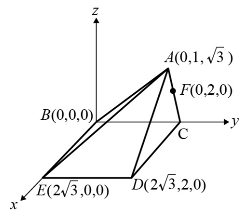
$\ge\left(\sqrt{16-3\left|\overrightarrow{b}\right|^2}+\left|\overrightarrow{b}\right|\right)^2$

，即 $\dfrac{64}{3}\ge\left(\sqrt{16-3\left|\overrightarrow{b}\right|^2}+\left|\overrightarrow{b}\right|\right)^2$，所以 $\sqrt{16-3\left|\overrightarrow{b}\right|^2}+\left|\overrightarrow{b}\right|\le\dfrac{8}{\sqrt{3}}$

第貳部分：非選擇題

1. (1) $\dfrac{2}{\sqrt{13}}$ (2) 1

【詳解】



(1)將五面體置於坐標平面上，並給予各點座標，先求出 E_{BFD}

$\overline{AE}=(2\sqrt{3},-1,-\sqrt{3}),\overline{BD}=(2\sqrt{3},2,0)=2(\sqrt{3},1,0)$ (1 分)

$\overrightarrow{n}\,//\,\overrightarrow{AE}\times\dfrac{1}{2}\overrightarrow{BD}=(\sqrt{3},-3,3\sqrt{3})$ (1 分)

$\therefore \overrightarrow{n}$ 取$(1,-\sqrt{3},3)$，又 $\overrightarrow{n_{xy}}=(0,0,1)$

$\cos\theta=\pm\dfrac{\overrightarrow{n}\cdot\overrightarrow{n_{xy}}}{\left|\overrightarrow{n}\right|\left|\overrightarrow{n_{xy}}\right|}=\pm\dfrac{3}{\sqrt{13}\times 1}=\pm\dfrac{3}{\sqrt{13}}$ (2 分)，

$\therefore \sin\theta=\dfrac{2}{\sqrt{13}}$ (1 分)

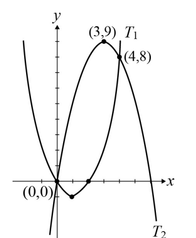
(2) $\overrightarrow{n}=(1,-\sqrt{3},3)$，可得 $E_{BDF}: x-\sqrt{3}y+3z=0$ (1 分)

又 $\overline{AC}:\begin{cases}x=0 \\ y=1+t \\ z=\sqrt{3}-\sqrt{3}t\end{cases}$,$0\le t\le 1$,代入E_{BDF} (2 分)

$0-\sqrt{3}-\sqrt{3}t+3\sqrt{3}-3\sqrt{3}t=0, t=\dfrac{1}{2}$

可得 $F\left(0,\dfrac{3}{2},\dfrac{\sqrt{3}}{2}\right)$(1分) $\Rightarrow \overline{AF}=\sqrt{\dfrac{1}{4}+\dfrac{3}{4}}=1$ (1 分)

2. (1) $\left(\dfrac{3}{2},-3\right)$ (2) $\dfrac{64}{3}$ (3) 2



【詳解】

(1) T_1：$y=x^2-2x=(x-1)^2-1$

$y'=2x-2$(1 分)

$2x-2=-2$，$x=0$，$y=0$

$2x-2=4$，$x=3$，$y=3$

$\left.\begin{matrix}L_1: m_1=-2,\text{過}(0,0)\Rightarrow 2x+y=0(1\text{分}) \\ L_2: m_2=4,\text{過}(3,3)\Rightarrow 4x-y=9(1\text{分})\end{matrix}\right\} \Rightarrow P\left(\dfrac{3}{2},-3\right)$ (1 分)

(2) $\begin{cases}y=x^2-2x \\ y=-x^2+6x\end{cases}\Rightarrow 2x^2-8x=0$，$x=0$ or 4

$\int_0^4\left[\left(-x^2+6x\right)-\left(x^2-2x\right)\right]dx$ (2 分)

$=\int_0^4\left(-2x^2+8x\right)dx=-\dfrac{2}{3}x^3+4x^2\Big|_0^4$ (1 分) $=-\dfrac{2}{3}\times 64+64=\dfrac{64}{3}$ (2 分)

(3) $\because f(x)=x^2-2x$，$g(x)=-x^2+6x$

x^2 係數皆爲 1，開口寬度一致(2 分)　∴連接(0 , 0)、(4 , 8)

得 $y=2x$，即可使面積平分，$\therefore k=2$(3 分)

【另解】

$\begin{cases}y=kx \\ y=x^2-2x\end{cases}\Rightarrow x=0$ 或 $k+2$

$\int_0^{k+2}\left(kx-(x^2-2x)\right)dx=\dfrac{32}{3}$

$\dfrac{-1}{3}x^3+\dfrac{k+2}{2}x^2\Big|_0^{k+2}=\dfrac{32}{3}$

$\dfrac{-1}{3}(k+2)^3+\dfrac{(k+2)^3}{2}=\dfrac{32}{3}$

$(k+2)^3=64$

$\therefore k=2$