

# 數學考科解析

考試日期：107 年 12 月 20~21 日

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	2	3	3	1	5	1	135	234	145	124	135	8	1	1
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
5	0	2	0	1	3	0	7	1	5	4	3	4	7	4
31	32	33	34											
1	5	1	8											

## 第壹部分：選擇題

### 一、單選題

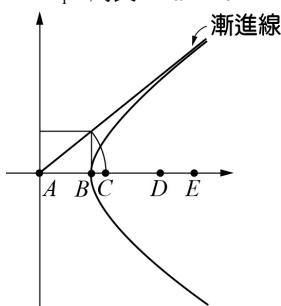
1. 令  $x$  為原分數平均， $y$  為新分數平均， $y = ax + b$

$$\begin{cases} 45a + b = 60 \\ |a| \cdot 15 = 10 \Rightarrow (a, b) = (\frac{2}{3}, 30) \\ a > 0, b > 0 \end{cases}$$

$$60 \times \frac{2}{3} + 30 = 70$$

2.  $2b = 6$ ， $b = 3$ ， $c = 4$ ， $a = 5$

$$\triangle AF_1B \text{ 周長} = 4a = 20$$

3. 

$$4. \frac{1 \cdot (2^{30} - 1)}{2 - 1} = 2^{30} - 1$$

$$\log 2^{30} = 30 \log 2 = 30 \times 0.3010 = 9.03$$

$$\therefore 2^{30} = 1.0 \times 10^9$$

5. 設小明射門  $n$  次

$$1 - (\frac{1}{3})^n > 0.99 \Rightarrow (\frac{1}{3})^n < 0.01 \Rightarrow 3^n > 100$$

$$\text{又 } n = 5 \text{ 時， } 3^5 = 243$$

$$\therefore n \text{ 取 } 5$$

$$6. \begin{cases} \alpha + \beta + \gamma = 2 \\ \alpha\beta + \beta\gamma + \gamma\alpha = -3 \\ \alpha\beta\gamma = 4 \end{cases}$$

$$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$4 = \alpha^2 + \beta^2 + \gamma^2 - 6, \alpha^2 + \beta^2 + \gamma^2 = 10$$

$$\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = (\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha) = 2 \cdot (10 + 3) = 26$$

7.  $4 \times 3 \times 2 - 1 \times 3 \times 1 - 3 \times 1 \times 2 + 1 \times 1 \times 1 = 16$

紫 蛋 非 雞 紫 雞 蛋

白  
飯

### 二、多選題

$$8. (1) \det(A) = \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} = 7$$

$$(2) B = \frac{1}{7} \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{bmatrix}$$

- (3)  $A, B$  互為反矩陣

$$\therefore AB = BA = I$$

$$\text{得 } (A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$$

$$(4) A + 3C = \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 6 \\ -3 & 12 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -2 & 14 \end{bmatrix}$$

$$(5) AC = \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -2 & 10 \end{bmatrix}$$

$$9. (1) \begin{cases} x - y = 0 \\ 2x + y = 3 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases}$$

$$(2)(3) \because \frac{|1+1|}{\sqrt{1+1}} = \sqrt{2} \therefore \text{相切}$$

$$\therefore (t, t) \text{ 代入 } C: (t-1)^2 + (t+1)^2 = 2, t = 0$$

$$\therefore \text{切點 } (0, 0)$$

$$(4) \frac{|2-1-3|}{\sqrt{4+1}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$(5) \overline{AB} = 2\sqrt{(\sqrt{2})^2 - (\frac{2}{\sqrt{5}})^2} = 2\sqrt{\frac{6}{5}} = \frac{2\sqrt{30}}{5}$$

$$10. f(x) = (x-1)^2(x-2)Q(x) + 2x^2 + 3x + 4$$

$$f(1) = 2 + 3 + 4 = 9, f(2) = 8 + 6 + 4 = 18$$

$$f(3) = 4Q(3) + 31$$

$$f(-3) = -80Q(-3) + 13$$

$$20f(3) - f(-3) = 80Q(3) + 620 + 80Q(-3) - 13 = 607$$

$$\begin{array}{r} 1 \quad -2 \quad 1 \quad \overline{) 2 \quad 3 \quad 4} \\ \underline{2 \quad -4 \quad 2} \phantom{0} \\ 7 \quad 2 \end{array} \quad \begin{array}{r} 1 \quad -3 \quad 2 \quad \overline{) 2 \quad 3 \quad 4} \\ \underline{2 \quad -6 \quad 4} \phantom{0} \\ 9 \quad 0 \end{array}$$

- 11.

	甲	乙	丙	丁	戊	
$x$	2	3	4	5	6	$\mu_x = 4$
$y$	4	4	6	10	6	$\mu_y = 6$
$x - \mu_x$	-2	-1	0	1	2	
$y - \mu_y$	-2	-2	0	4	0	
$(x - \mu_x)(y - \mu_y)$	4	2	0	4	0	合計 = 10
$(x - \mu_x)^2$	4	1	0	1	4	合計 = 10
$(y - \mu_y)^2$	4	4	0	16	0	合計 = 24

$$(1)(2) r = \frac{10}{\sqrt{10} \sqrt{24}} = \frac{5}{2\sqrt{15}} \div 0.64$$

$$(3) m = \frac{10}{10} = 1$$

$$(4) y - 6 = 1 \cdot (x - 4) \Rightarrow y = x + 2$$

$$(5) x = 8 \text{ 代入 } y = x + 2 = 10$$

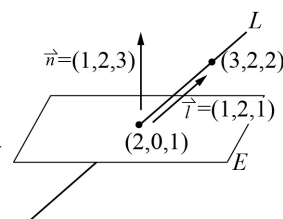
12. (1)  $(1+t, -2+2t, t)$  代入  $E$

$$1+t-4+4t+3t=5, t=1$$

$$\text{交點為 } (2, 0, 1)$$

$$(2) \frac{|3+4+6-5|}{\sqrt{1+4+9}} = \frac{8}{\sqrt{14}} = \frac{4\sqrt{14}}{7}$$

$$(3) \vec{n} \cdot \vec{\ell} = 1+4+3=8$$



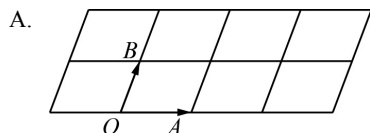
$$(4) \frac{1+4+3}{14}(1,2,3) = \left(\frac{4}{7}, \frac{8}{7}, \frac{12}{7}\right)$$

$$(5) (1,2,1) - \left(\frac{4}{7}, \frac{8}{7}, \frac{12}{7}\right) = \left(\frac{3}{7}, \frac{6}{7}, -\frac{5}{7}\right)$$

投影直線方向向量取  $(3, 6, -5)$

$$\frac{x-2}{3} = \frac{y}{6} = \frac{z-1}{-5}$$

## 第貳部分：選填題



$$\vec{OA} = (1, 2), \vec{OB} = (3, 7)$$

$$8 \times \left| \begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix} \right| = 8$$

B.  $\frac{1}{6 \times 5 \times 5} = \frac{1}{150}$

C.  $a_1 = 10$

$$a_1 + a_2 = 4 \Rightarrow a_2 = -6$$

$$a_2 + a_3 = 6 \Rightarrow a_3 = 12$$

$$a_3 + a_4 = 8 \Rightarrow a_4 = -4$$

$$a_4 + a_5 = 10 \Rightarrow a_5 = 14$$

$$a_5 + a_6 = 12 \Rightarrow a_6 = -2$$

$$a_6 + a_7 = 14 \Rightarrow a_7 = 16$$

$$a_7 + a_8 = 16 \Rightarrow a_8 = 0$$

$$a_8 + a_9 = 18 \Rightarrow a_9 = 18$$

$$a_9 + a_{10} = 20 \Rightarrow a_{10} = 2$$

$$a_{10} + a_{11} = 22 \Rightarrow a_{11} = 20$$

<另解>

$$a_{2n+1} = 10 + 2n, a_{2n} = 2n - 8$$

$$\therefore a_{11} = 10 + 10 = 20$$

D.

	甲	乙	丙
原	$x$	$y$	$z$
第一輪	$x - y - z$	$2y$	$2z$
第二輪	$2(x - y - z)$	$2y - (x - y - z) - 2z$	$4z$
第三輪	$4(x - y - z) = 80$	$4y - 2(x - y - z) - 4z = 80$	$4z - 2(x - y - z) - 2y + (x - y - z) + 2z = 80$

$$\Rightarrow (x, y, z) = (130, 70, 40)$$

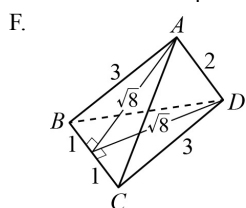
E.  $\overline{AC}^2 = 4 + 16 - 16 \cos \theta = 4 + 9 + 12 \cos \theta$

$$\cos \theta = \frac{1}{4}, \sin \theta = \frac{\sqrt{15}}{4}$$

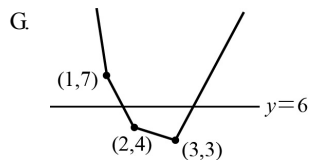
$$\overline{AC}^2 = 16, \overline{AC} = 4$$

$$\square ABCD = \frac{1}{2} \times 2 \times 4 \times \frac{\sqrt{15}}{4} + \frac{1}{2} \times 2 \times 3 \times \frac{\sqrt{15}}{4}$$

$$= \frac{7\sqrt{15}}{4}$$



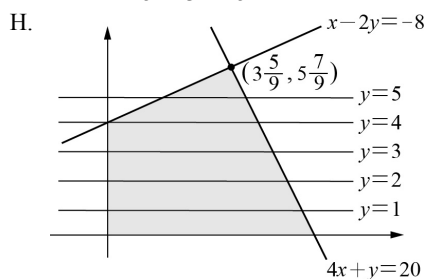
$$\cos \theta = \frac{8+8-4}{2 \times \sqrt{8} \times \sqrt{8}} = \frac{3}{4}$$



$$\alpha - 1 + \alpha - 2 + 3\alpha - 9 = 6, \alpha = \frac{18}{5}$$

$$\beta - 1 - \beta + 2 - 3\beta + 9 = 6, \beta = \frac{4}{3}$$

$$\alpha + \beta = \frac{18}{5} + \frac{4}{3} = \frac{74}{15}$$



$$y = 1 \Rightarrow x = 1, 2, 3, 4$$

$$y = 2 \Rightarrow x = 1, 2, 3, 4$$

$$y = 3 \Rightarrow x = 1, 2, 3, 4$$

$$y = 4 \Rightarrow x = 1, 2, 3, 4$$

$$y = 5 \Rightarrow x = 2, 3$$

$$4 \times 4 + 2 = 18 \text{ (個)}$$

